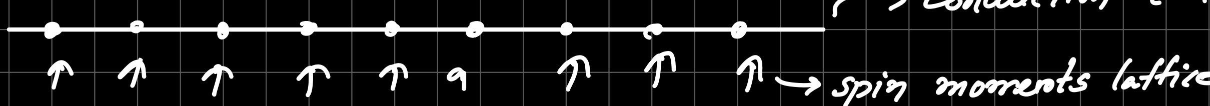
Physics 211C: Solid State Physics

Instructor: Prof. Tarun Grover — Lecture 10

Topic: Heavy Fermi Liquids

extension of kondo problem

single impurity Kondo $\quad \dfrac{dJ}{dl} = J^2$



Expt data :-

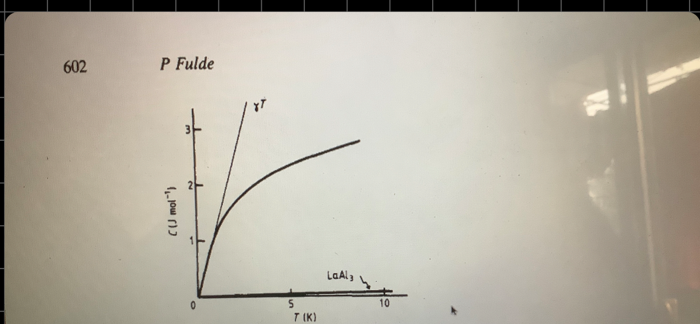
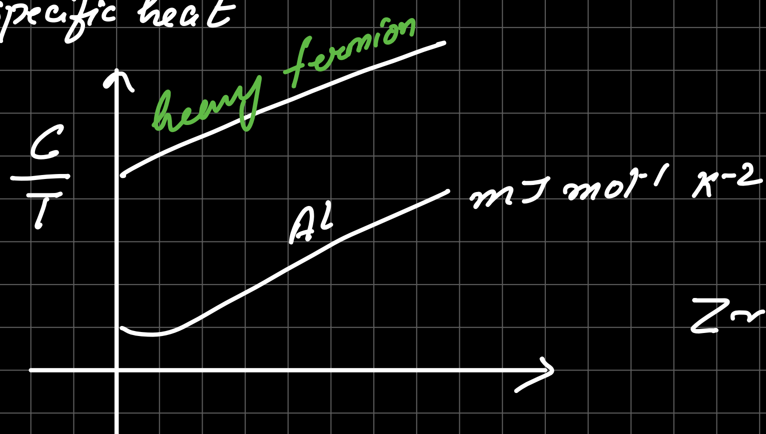


Figure 1. Schematic plot of the specific heat $C(T)$ for $CeAl_3$. For comparison we show also the results for the related compound $LaAl_3$. Usually one plots $C(T)/T$ against T^2 but the present form shows better that the high density of low-lying excitations is limited to low T.

γ (at $c=1$) is very large in these materials

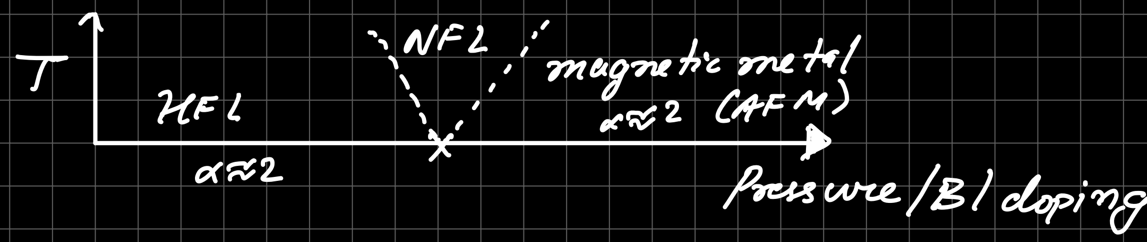
↓ specific heat coefficient

Large specific heat



$Z \sim \dfrac{m}{m^*} <<< 1$

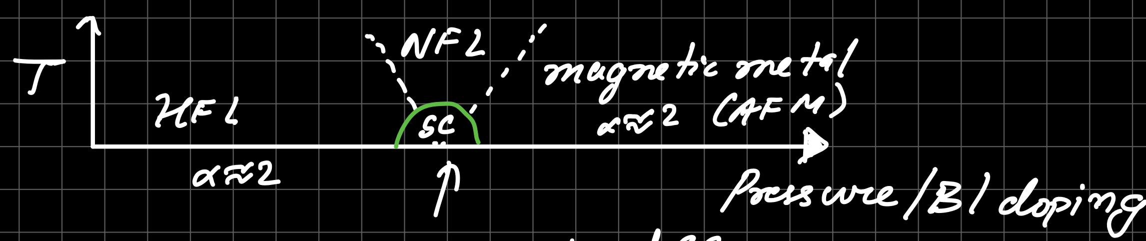
HFL generally hosts a QCP



$\rho \sim T^{\alpha}$

Despite large dos, en. required to eject an $e^{\ominus}$ is large.

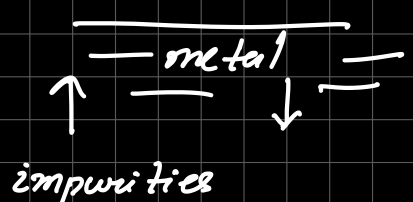
USC can be found at QCP



$\rho \sim T^{\alpha}$

unconventional SC
(nodes, $\frac{T_c}{T_F}$ is large $\sim 10^{-2}, \frac{1}{50}$)

Doniach picture of competition b/w Kondo & magnetic ordering



eff int b/w spins (mediated by FL)

$$\sum_{x x'} S(x) S(x') \, J(x-x')$$

$$J(r) \sim \frac{\cos(2k_F r)}{r^3} \quad \text{in } d=3$$

$$h_{eff}(x') \sim \int_x \chi(x-x') \, S(x)$$

magnetic susceptibility

} because in linear response

$$\underbrace{c^{\dagger} \vec{\sigma} c}_{h} \cdot \vec{S}$$

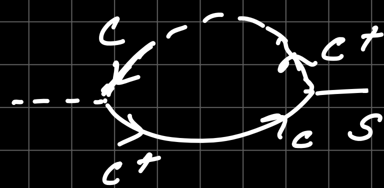
generates an eff. coupling

of the type

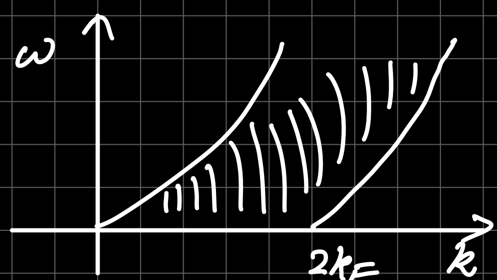
$\chi_{xx'}\ S_x S_{x'}$

$$\chi \sim \left[\sum_{x} c^{\dagger}\sigma c \, , \sum_{x'} c^{\dagger}\sigma c\right]$$

diagrammatically



$\chi(2k_F) \to \infty$ due to p-h excitat^{ns} having

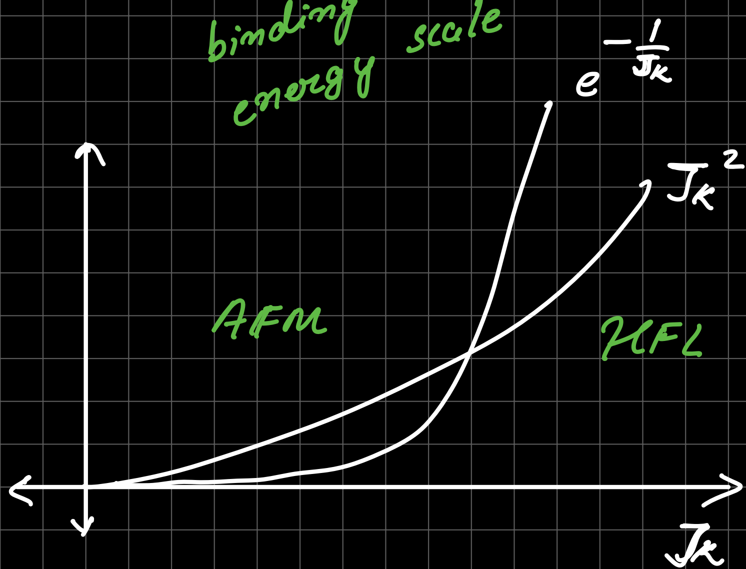


$$H_{eff} \simeq J_K\, c^{\dagger}\vec{\sigma}c\cdot\vec{S} + \frac{\cos(2k_F r)}{|r-r'|^3}\, s(r)\cdot s(r')\, J_K^2$$

2 energy scales

$$T_K \sim e^{-\frac{1}{J_K N(0)}}, \quad J_K^2 \Rightarrow \text{competition b/w these scales}$$

binding energy scale



Brian Maple → great expt contributions (@UCSD)

Harry Suhl

* Large SOC $\rightarrow$ leads to large spin # e.g. $J = 7/2$

$$S = f^{\dagger} \Gamma f \qquad (2J+1) \text{ levels}$$

Effective model to quantitatively understand large m^*

$$\mathcal{H} = \underbrace{\sum_{k\sigma} \epsilon_k c^{\dagger}_{k\sigma} c_{k\sigma}}_{\mathcal{H}_0}$$

$$+ \frac{J_K}{2} \sum_{r} \left(c_r^{\dagger} \vec{\sigma} c_r \right) \cdot \vec{S}_r \rightarrow \text{spin } 1/2$$

$$\vec{S}_r = \frac{f^{\dagger} \vec{\sigma} f}{2} \rightarrow \text{spinons} \rightarrow \text{fermions}$$

f = fermion = $\begin{bmatrix} f_{\uparrow} \\ f_{\downarrow} \end{bmatrix} \Rightarrow$ S^2 fixed $\sum_{\sigma} f_{\sigma}^{\dagger} f_{\sigma} = 1$

don't carry charge

spin $\frac{1}{2}$, charge 0 objects

$$\frac{J_K}{2 \times 2} c_{\alpha}^{\dagger} \sigma^{a}_{\alpha\beta} c_{\beta} f_{\gamma}^{\dagger} \sigma^{a}_{\gamma\delta} f_{\delta}$$

$$\sigma^{a}_{\alpha\beta} \sigma^{a}_{\gamma\delta} = 2 \delta_{\alpha\delta} \delta_{\beta\gamma} - \delta_{\alpha\beta} \delta_{\gamma\delta}$$

$$\Downarrow$$

$$-\frac{J_K}{2} c_{\alpha}^{\dagger} f_{\alpha} f_{\beta}^{\dagger} c_{\beta} + c^{\dagger} c$$

Main Idea:- since we seek HFL, we have guidance on what mean field to do.
(why does mean field work? $\Rightarrow$ "Argument")

Action:

$$S = \int d\tau \left[\bar{c} \partial_{\tau} c - \sum_{k} \epsilon_k \bar{c}_{\sigma}(k,\tau) c_{\sigma}(k,\tau) \right]$$

$$+\int d\tau \left[\bar{f}_\sigma \partial_\tau f_\sigma - i\, \underbrace{a_0(r,\tau)} \left(\bar{f}_\sigma(r,\tau) f_\sigma(r,\tau) - 1 \right) \right]$$

lagrange multiplier that ensures
$\sum_\sigma f_\sigma^\dagger f_\sigma = 1$

$$+\int d\tau \sum_r -\frac{J_K}{2} \left[\bar{c}_\alpha f_\alpha \bar{f}_\beta c_\beta \right]$$

$$Z = \int D[\bar{c}c]\, D[\bar{f}f]\, \underbrace{D[a_0]}\, e^{-S}$$

just leads to $\delta(\bar{f}_\sigma f_\sigma - 1)$

Other terms allowed by symmetries $\Rightarrow$ $+ \dots \bar{f}_r f_{r'} e^{i a_{rr'}}$

$a_{rr'}$: gauge fields (redudancy in description of f)

"Spinon-bandstructure"?

$+ (\nabla \times a)^2$ $\downarrow$ kinetic en. term for gauge field

Symmetries:

Physical syms (actual) : ① Charge conservation

$\sum_r c_r^\dagger c_r$ is allowed

$c \to c e^{i\theta}$

② $SU(2)$ spin rotation:-

$c \to \mathcal{U} c \qquad \mathcal{U} \in SU(2)$

$f \to \mathcal{U} f$

$S \to \mathcal{U}^\dagger S\, \mathcal{U}$

Redundant invariants (gauge invariance) : $f \to f e^{i\theta(r,\tau)}$

$a_0 \to a_0 + \partial_\tau \theta$ (0^{th} comp. of gauge field)

$\bar{f}\partial_\tau f - ia_0 \bar{f} f \longrightarrow$ remains invariant

$a_{rr'} \to a_{rr'} + \theta_r - \theta_{r'}$ $\quad (\bar{f}_r f_{r'} e^{-ia_{rr'}})$

[Long discussion of spinon fermi surfaces]

Now we use knowledge of gauge theory

deconfined phase:

Higgs phase: condense a field that carries gauge charge

$\langle f \rangle$?

$\langle c_\sigma^\dagger f_\sigma \rangle \to$ charge 1 object
$= b$?

$\langle c^\dagger c^\dagger \rangle$

(Condensing a charge 2 object, doesn't screen charge 1 object
$\Downarrow$
no Gauge fields at low energies)

$\Downarrow$

$U(1)$ gauge field
$\downarrow$ if we condense a charge 2 obj.
low. en. theory looks a little like

$$V = \frac{J_K}{2} \langle c_{r\sigma}^\dagger f_{r\sigma} + h.c. \rangle$$

$$\mathcal{H}_{MF} = \sum_{k\sigma} \varepsilon_k c_{k\sigma}^\dagger c_{k\sigma}$$

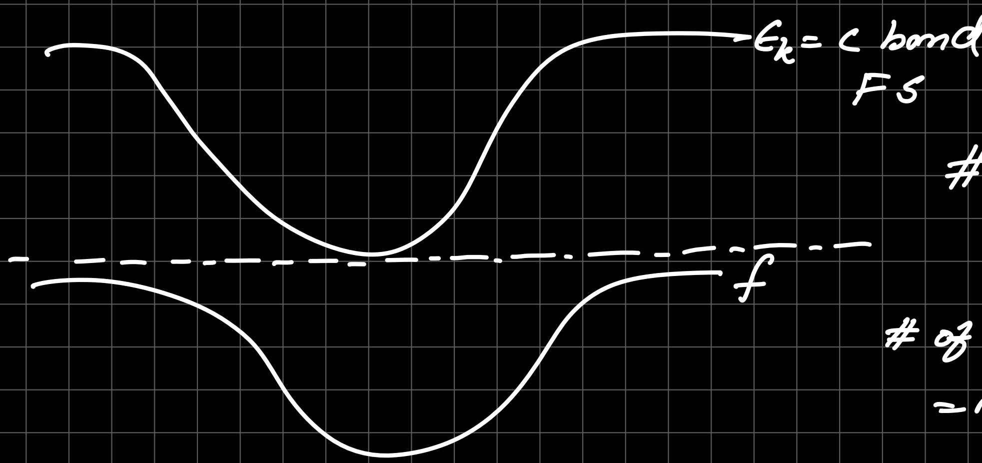
$$+ \mu_f \sum_{k\sigma} f_{k\sigma}^\dagger f_{k\sigma}$$

impose $f^\dagger f$ on an avg level

$$- V \sum_k [c_{k\sigma}^\dagger f_{k\sigma} + h.c.]$$

How did we get to this mean ???

field?



\# of spins = # of lattice sites

$= N_s$

\# of $e^{\ominus}$

$= N_c < N$

no spinons here (higgs phase of a Gauge theory)